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STOCHASTIC SYSTEMS

A Decomposition–Autocompensation Method for Signal Recognition Based on the Principles of Continuity, Invariance, Multiplication, and Ranking with Regular and Irregular Interferences

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Abstract—Considering the principles of continuity, invariance, multiplication, and ranking, we develop a novel optimal signal recognition method under essential a priori uncertainty, with application to real-time information-measuring systems. By assumption, in addition to random noise with an unknown distribution law but a given correlation matrix, the observation equation may contain a regular interference with an analytical finite-spectral representation and an irregular interference without any effective probabilistic model. The latter interference can be described only by introducing some numerical characteristics and constraints confirmed by the operation practice of a particular system. This method is invariant to the above interferences, does not require traditional state-space expansion, and ensures the decomposition of the computational procedure. We analyze random and methodological errors as well as the computational effect achieved. An illustrative example is given.

Keywords: essential a priori uncertainty, regular interference, irregular interference, sample clogging factor, continuity principle, invariance principle, multiplication principle, ranking principle, spectral coefficients, optimality criterion, decomposition, recognition algorithms

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1. INTRODUCTION

For a modern information-measuring system (hereinafter referred to as a/the system), signal recognition is one of the most important problems; it includes signal detection, discrimination, resolution, smoothing, parameter estimation, etc. [1–18].

In the sense of applications, of major interest are real-time systems under essential uncertainty and strict requirements to recognition quality, not only on the average (over an entire ensemble of realizations) but also in a single case (for one fixed sample of measurements). The matter concerns a system with quite high risks (losses) from an incorrect decision in such a case.

Essential uncertainty arises, e.g., when real measurements are subjected to random noise with an unknown distribution and, moreover, to regular and irregular interferences. In this case, the noise information is limited only by its correlation matrix; the regular interference information, by a given finite set of its basis functions; the irregular interference information, by some numerical characteristics and constraints for a particular system. For example, the sample clogging factor can be one such characteristic.

Essential uncertainty is often eliminated using the least squares (LS) method or one of its modifications in a simple or extended version [5, 7–9, 14–18]. By the well-known Gauss–Markov

theorem, this method yields the best estimates on average; for example, see [5, p. 34]. Less common methods include maximum posterior probability density, maximum likelihood, linear and nonlinear filtering, and some others [1–7, 11, 14]. As a rule, these methods require a rather large amount of a priori statistical information and are difficult to implement in real time (especially when expanding the state space). In addition, they are often unstable (in computational terms) and have poor convergence in iterative calculations, e.g., in the case of unsuccessfully assigned initial conditions or ravine-type objective functions. Such methods are most often used at stages related to mathematical modeling when justifying the potential capabilities of the system. In the single cases mentioned (when one fixed sample is involved), the estimation results can cause significant losses, especially under an irregular interference.

The method of generalized invariant-unbiased estimation (GIUE) [18, 19] was developed for any linear numerical characteristics of useful signals (e.g., spectral coefficients, derivatives of different orders, integrals, smoothed values, etc.) under a regular interference (sometimes called signal-like, systematic, singular, or dynamic) without state space expansion. GIUE autocompensates for a regular interference, smoothens random noise, and yields optimal estimates with the minimum trace of the correlation matrix of estimation errors; no a priori information about the distribution law is required, and it suffices to know only the correlation matrix for a given noise. The method ensures the maximum possible decomposition of the computational procedure, which leads to the inversion of matrices of significantly lower dimensions compared to the extended least squares (XLS) method.

However, the GIUE capabilities are significantly limited if measurements contain both regular and irregular interferences (e.g., in the form of separate single pulses or packets of different-shape pulses with unknown parameters). An irregular interference can occur both on the entire observation interval and on separate, a priori unknown, parts of this interval and is poorly formalized (e.g., within finite-analytical spectral analysis). As a rule, an attempt to introduce additional basis functions for describing an irregular interference sharply increases dimension and causes significant errors. Under essential uncertainty and real observational conditions, expanding the spectral composition of the total interference (regular plus irregular interferences) requires inverting ill-conditioned matrices of too high dimensions, which strongly distorts the results of measurement processing within GIUE.

Below we develop a novel signal recognition method inheriting all the GIUE advantages without state space expansion. Therefore, it is possible to implement the corresponding algorithms in real time for the system under uncertainty. This method provides a trade-off between average estimation (based on the conditions of unbiasedness, invariance to a regular interference, and the minimum trace of the correlation matrix of errors) and partial estimation (for a fixed sample) considering the minimum influence of an irregular interference on the resulting estimate.

To avoid rather cumbersome calculations, the presentation below will mainly focus on the problem of estimating the linear parameters of signals. However, we will also discuss appropriate generalizations and practical recommendations for extending the results to other problems related to signal recognition, including those with nonlinear parameters (by analogy with [19]).

2. MATHEMATICAL FORMULATION OF THE RECOGNITION PROBLEM IN THE CASE OF ESSENTIAL A PRIORI UNCERTAINTY

On a given time interval $[0, T]$, we introduce a basic grid $\mathrm{C}_0 = \left\{t_n, n = \overline{1, N_0}\right\}$ of nodes and write the following observation equation on this grid:

$$\mathbf{H}_0 = \mathbf{S}_0 + \mathbf{\Theta}_0 + \mathbf{D}_0 + \mathbf{\Xi}_0, \tag{1}$$

where $\mathbf{H}_0 = \left[h_n, n = \overline{1, N_0}\right]^{\mathrm{T}}$, $\mathbf{S}_0 = \left[s_n(\mathbf{A}), n = \overline{1, N_0}\right]^{\mathrm{T}}$, $\mathbf{\Theta}_0 = \left[\theta_n(\mathbf{B}_\theta), n = \overline{1, N_0}\right]^{\mathrm{T}}$, $\mathbf{D}_0 = \left[q_n d_n(\mathbf{B}_d), n = \overline{1, N_0}\right]^{\mathrm{T}}$, and $\mathbf{\Xi}_0 = \left[\xi_n, n = \overline{1, N_0}\right]^{\mathrm{T}}$ are the vectors of samples of an input observation $h(t)$, a useful signal $s(t, \mathbf{A})$, a regular interference $\theta(t, \mathbf{B}_\theta)$, an irregular interference $d(t, \mathbf{B}_d)$, and noise $\xi(t)$, respectively, $h_n = h(t_n)$, $s_n(\mathbf{A}) = s(t_n, \mathbf{A})$, $\theta_n(\mathbf{B}_\theta) = \theta(t_n, \mathbf{B}_\theta)$, $d_n(\mathbf{B}_d) = d(t_n, \mathbf{B}_d)$, and $\xi_n = \xi(t_n)$; $\mathbf{A} = \left[a_m, m = \overline{1, M_s}\right]^{\mathrm{T}}$, $\mathbf{B}_\theta = \left[b_{\theta r}, r = \overline{1, M_\theta}\right]^{\mathrm{T}}$, and $\mathbf{B}_d = \left[b_{dr}, r = \overline{1, M_d}\right]^{\mathrm{T}}$ are the unknown vector spectral coefficients of the linear decompositions of the signal, regular, and irregular interference, respectively; finally, $q_n \in \{0, 1\}$ is the indicator of zero and nonzero irregular interference samples, where $\sum_{n=1}^{N_0} q_n = M_d$ and the parameter M_d corresponds to the real number of nonzero irregular interference samples in the observation equation (1).

Let $\{\overline{t_1}, \overline{t_2}, \ldots, \overline{t_{M_d}}\}$ denote the set of nodes associated with nonzero irregular interference samples, where $\overline{t_m} \in \{t_1, t_2, \ldots, t_{N_0}\}$ and $\overline{t_{m+1}} > \overline{t_m}$. To describe the irregular interference, we use the following approach. Let $K_{\max} \in \{0, 1, \ldots\}$ be the maximum possible number of nodes associated with the irregular interference, i.e., $M_d \leqslant K_{\max}$. We require the condition

$$N_{\min} + K_{\max} \leqslant N_0, \tag{2}$$

where $N_{\min}$ is the minimum number of input observation samples sufficient to qualitatively recognize the useful signal in the absence of regular and irregular interferences.

We will consider different time grids of nodes of length N for which

$$N_{\min} \leqslant N \leqslant N_0. \tag{3}$$

With this descriptive approach to irregular interferences, their nonzero samples can take arbitrary values (including anomalous ones) as well as be scattered (single) or concentrated (in packets). In physical channels, such interferences may correspond, e.g., to pulses of various shapes, durations, and intensities. These interferences are often due to various transients, switching, hashing, natural and artificial disturbance, etc., and may be masked by signal and noise. Nonzero irregular interference samples can be arbitrarily located on the segment $[0, T]$, and there exists no universal and satisfactory model to describe them. The only way to consider irregular interferences is to impose some quantitative constraints (like (2) and (3)) matching the operation practice of a particular system.

For arbitrary values of $\mathbf{A}$, $\mathbf{B}_\theta$, and $\mathbf{B}_d$, we use the following linear finite-dimensional combinations (the signal and regular interference models widespread in practice):

$$s(t, \mathbf{A}) = \mathbf{A}^{\mathrm{T}} \mathbf{\Psi}(t), \tag{4}$$

$$\theta(t, \mathbf{B}_\theta) = \mathbf{B}_\theta^{\mathrm{T}} \mathbf{\Omega}_\theta(t), \tag{5}$$

$$d(t, \mathbf{B}_d) = \mathbf{B}_d^{\mathrm{T}} \mathbf{\Omega}_d(t), \tag{6}$$

where $\mathbf{\Psi}(t) = \left[\psi_m(t), m = \overline{1, M_s}\right]^{\mathrm{T}}$, $\mathbf{\Omega}_\theta(t) = \left[\omega_{\theta r}(t), r = \overline{1, M_\theta}\right]^{\mathrm{T}}$ and $\mathbf{\Omega}_d(t) = \left[\omega_{dr}(t), r = \overline{1, M_d}\right]^{\mathrm{T}}$ are given basis functions of the signal, regular, and irregular interference, respectively, $\omega_{dr}\left(\overline{t_r}\right) = 1$, and $\omega_{dr}(t) = 0$ for all $t \neq \overline{t_r}$.

Suppose that the extended functional basis $\{\mathbf{\Psi}(t), \mathbf{\Omega}_\theta(t), \mathbf{\Omega}_d(t)\}$ is linearly independent on the grid C_0 (by analogy with [18, 19]). The noise $\mathbf{\Xi}_0$ is characterized by zero mean and a correlation matrix $\mathbf{K}_{\mathbf{\Xi}_0}$.

Note that quite rare anomalous outliers of the noise $\xi(t)$ are further associated with irregular interferences, i.e., they are combined with the corresponding nonzero coordinates of the vector $\mathbf{D}_0$. Thus, in any sample $\mathbf{\Xi}_0$, all coordinates can be enclosed in gates of a size determined by one of the well-known rules (e.g., by the three-sigma rule).

For the recognition problem, we will consider two main subproblems as follows.

Subproblem 1 (the classical problem of estimating the useful signal parameters) is to construct an optimal estimate $\mathbf{A}^*$ of the vector $\mathbf{A}^*$ based on the algorithm that autocompensates for the regular and irregular interferences. *Subproblem 2* is to construct estimates $\mathbf{A}^*$ and $\mathbf{B}^*$ for the vectors $\mathbf{A}$ and $\mathbf{B}^*$, respectively, invariant to the irregular interference. In this case, the estimation of the signal parameters shall autocompensate for the regular interference and, vice versa, the estimation of the regular interference parameters shall autocompensate for the signal.

Based on (1)–(6), it is required to develop an optimal recognition method (for the two main subproblems above) under essential a priori uncertainty (using only the correlation matrix $\mathbf{K}_{\mathbf{\Xi}_0}$ as statistical information) without state space expansion. In both subproblems, optimality means that the estimates shall be unbiased, have the minimum trace of the correlation matrices of estimation errors, and provide a significant computational advantage over XLS and GIUE, in terms of decomposition and reduction in the amount of calculations and a gain in accuracy due to inverting matrices of smaller dimensions.

3. A GENERAL APPROACH TO SIGNAL RECOGNITION BASED ON THE PRINCIPLES OF CONTINUITY, MULTIPLICATION, AND RANKING

The approach proposed below involves the principle of a continuous dependence of recognition quality on the parameters of the time grid. In particular, the successive reduction of the volume of a sufficiently large grid (by removing certain nodes) leads to a smooth evolution of the accuracy of the resulting estimate. This effect also concerns a continuous change in the position of grid nodes on a given time interval.

Assume that a set of different reduced grids can be formed from the basic grid C_0 and its variants obtained by removing some nodes: $\left\{\mathrm{C}_j, j=\overline{1,J-1}\right\}$ (where $J \geqslant 2$, $\mathrm{C}_j = \left\{t_{jn}, n=\overline{1,N_j}\right\}$, $N_j < N_0$, $t_{jn} \in \mathrm{C}_0$, $t_{jn} \neq t_{jk}$ $\forall n \neq k$, $n,k \in \overline{1,N_j}$, $t_{j,n+1} > t_{jn}$).

We represent the entire family of grids $\mathrm{C} = \{\mathrm{C}_0, \mathrm{C}_1, \ldots, \mathrm{C}_{J-1}\}$, including C_0, as

$$\mathrm{C} = \mathrm{C}^{\#} \cup \mathrm{C}^{\wedge} \cup \mathrm{C}^{\&}. \tag{7}$$

The symbol $\mathrm{C}^{\#}$ indicates the set of grids without nodes associated with nonzero irregular interference samples. Next, the symbol $\mathrm{C}^{\wedge}$ stands for the set of grids that may contain nodes associated with normal irregular interference samples. These are the irregular interference samples that, all together, weakly affect the estimation results. Finally, the symbol $\mathrm{C}^{\&}$ denotes the set of grids that may contain nodes associated with both normal and abnormal irregular interference samples that, all together, devalue the estimation results. The sets $\mathrm{C}^{\#}$ and $\mathrm{C}^{\wedge}$ will be called admissible and the set $\mathrm{C}^{\&}$ inadmissible.

The main idea of the method is that by handling $\mathrm{C}^{\#}$ and $\mathrm{C}^{\wedge}$, one can obtain estimates invariant (or almost invariant) to regular and irregular interferences (the invariance principle). For this purpose, we assign to each grid C_j, $j = \{0, 1, \ldots, J-1\}$, the observation equation

$$\mathbf{H}_j = \mathbf{S}_j + \mathbf{\Theta}_j + \mathbf{D}_j + \mathbf{\Xi}_j \tag{8}$$

and the GIUE-optimal estimates

$$\begin{cases} \mathbf{A}_j^* = \mathbf{P}_j^{\mathbf{A}} \mathbf{H}_j, \quad s_j^*\left(t, \mathbf{A}_j^*\right) = \left(\mathbf{A}_j^*\right)^{\mathrm{T}} \mathbf{\Psi}(t), \\ \mathbf{B}_{\theta j}^* = \mathbf{P}_{\theta j}^{\mathbf{B}} \mathbf{H}_j, \quad \theta_j^*\left(t, \mathbf{B}_{\theta j}^*\right) = \left(\mathbf{B}_{\theta j}^*\right)^{\mathrm{T}} \mathbf{\Omega}_\theta(t), \end{cases} \tag{9}$$

where $\mathbf{H}_j$ is the vector of the input observation samples $\mathbf{H}_0$ corresponding to the reduced grid C_j, and $\mathbf{P}_j^{\mathbf{A}}$ and $\mathbf{P}_{\theta j}^{\mathbf{B}}$ are the linear decomposition estimation matrices:

$$\begin{cases} \mathbf{P}_j^{\mathbf{A}} = \left[\mathbf{\Lambda}_{\theta j}^{\mathbf{A}}\mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Psi}_j\left(\mathbf{\Psi}_j^{\mathrm{T}}\mathbf{\Lambda}_{\theta j}^{\mathbf{A}}\mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Psi}_j\right)^{-1}\right]^{\mathrm{T}}, \\ \mathbf{P}_{\theta j}^{\mathbf{B}} = \left[\mathbf{\Lambda}_{\theta j}^{\mathbf{B}}\mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Omega}_{\theta j}\left(\mathbf{\Omega}_{\theta j}^{\mathrm{T}}\mathbf{\Lambda}_{\theta j}^{\mathbf{B}}\mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Omega}_{\theta j}\right)^{-1}\right]^{\mathrm{T}}, \end{cases} \tag{10}$$

$$\begin{cases} \mathbf{\Lambda}_{\theta j}^{\mathbf{A}} = \mathbf{E}_{Nj} - \mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Omega}_{\theta j}\left(\mathbf{\Omega}_{\theta j}^{\mathrm{T}}\mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Omega}_{\theta j}\right)^{-1}\mathbf{\Omega}_{\theta j}^{\mathrm{T}}, \\ \mathbf{\Lambda}_{\theta j}^{\mathbf{B}} = \mathbf{E}_{Nj} - \mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Psi}_j\left(\mathbf{\Psi}_j^{\mathrm{T}}\mathbf{K}_{\mathbf{\Xi}j}^{-1}\mathbf{\Psi}_j\right)^{-1}\mathbf{\Psi}_j^{\mathrm{T}}. \end{cases} \tag{11}$$

In formulas (9)–(11), $\mathbf{\Psi}_j = [\psi_{jnm}, n = \overline{1,N_j}, m = \overline{1,M_s}]$ and $\mathbf{\Omega}_{\theta j} = [q_{jn}\omega_{\theta jnr}, n = \overline{1,N_j}, r = \overline{1,M_{\theta j}}]$ are the basis matrices of the signal $\mathbf{S}_j$ and interference $\boldsymbol{\theta}_j$, respectively, $\psi_{jnm} = \psi_m(t_{jn})$ and $\omega_{\theta jnr} = \omega_{\theta r}(t_{jn})$; $\mathbf{E}_{Nj}$ is an identity matrix corresponding to the dimension of the vector $\mathbf{H}_j$.

If $\mathrm{C}_j \in \mathrm{C}^{\#}$, these estimates will match the corresponding conditions of minimality, unbiasedness, and invariance [18, 19] (i.e., will be optimal). If $\mathrm{C}_j \in \mathrm{C}^{\wedge}$, we obtain quasi-optimal estimates; in the case $\mathrm{C}_j \in \mathrm{C}^{\&}$, the estimation errors may become unacceptably large.

The multiplication principle consists in the possibility of forming a set of partial estimates corresponding to the family of grids $\mathrm{C} = \{\mathrm{C}_0, \mathrm{C}_1, \ldots, \mathrm{C}_{J-1}\}$ satisfying conditions (2) and (3).

Consider subproblem 1. For each pair of grids C_j and C_m, we form the scalar residuals

$$\Delta_{\mathbf{A}jm}(t) = s(t, \mathbf{A}_j^*) - s(t, \mathbf{A}_m^*) = [(\mathbf{A}_j^*) - (\mathbf{A}_m^*)]^{\mathrm{T}}\mathbf{\Psi}(t), \quad j,m = \overline{0, J-1},\ j > m,$$

and their norms $\Delta_{\mathbf{A}jm} = \|\Delta_{\mathbf{A}jm}(t)\|$, $j,m = \overline{0, J-1}$ (any norm of the corresponding functional space). Based on the norms, we construct a variational series $\Delta_{\mathbf{A}[v]}$, $v = \overline{1, (J^2-J)/2}$ (a monotonically increasing sequence of scalar residuals), which fully characterizes the quality of all grids used. If at least one grid in a pair C_j, C_m belongs to the set $\mathrm{C}^{\&}$, the corresponding element $\Delta_{\mathbf{A}[v]}$ will be in the tail of the variational series. The initial elements of this series will be associated with the pairs C_j, C_m corresponding to the sets $\mathrm{C}^{\#}$ and $\mathrm{C}^{\wedge}$. Thus, under the above constraints, guaranteed clustering is performed for some elements $\Delta_{\mathbf{A}[v]}$ of the variational series in a sufficiently small neighborhood $(0, \delta_{\mathbf{A}})$, where $\delta_{\mathbf{A}} > 0$ denotes the truncation parameter of the series. For a given grid C_0, the constant $\delta_{\mathbf{A}}$ is chosen in advance for a particular system, considering the analytical expressions for the random and methodological errors of the GIUE procedure [18, 19], when planning a measurement experiment [5].

All the pairs of grids not satisfying the condition

$$\Delta_{\mathbf{A}[v]} < \delta_{\mathbf{A}} \tag{12}$$

are rejected. This is the ranking principle mentioned above.

We introduce several important notions.

Definition 1. A pair of grids C_j, C_m is said to be admissible if condition (12) holds.

Definition 2. An arbitrary irregular interference sample is said to be anomalous if its corresponding node, present in any grid of the set C, violates condition (12). Otherwise, the irregular interference sample is said to be normal.

Definition 3. A group of two or more samples is said to be anomalous (even if each separate sample is normal) when the corresponding nodes, present in any grid of the set C, violate condition (12). Otherwise, the group of irregular interference samples is said to be normal.

Definition 4. The sample clogging factor $\mathbf{H}_0$ is the number $k_d = 100\,(K_{\max}/N_0)$, expressed as a percentage.

Since all grids of the set $\mathrm{C}^{\&}$ violate condition (12), they are automatically rejected. A strict criterion for the anomalousness of input observation samples $\mathbf{H}_0$ sounds as follows: a sample is anomalous if it never occurs in pairs of grids satisfying condition (12). This criterion can be somewhat relaxed by considering the rare presence of potentially anomalous samples in the pairs specified. This is especially relevant in cases of ambiguity about an appropriate class (normal or anomalous) for a particular sample. Anyway, the criterion allows rejecting both anomalous and potentially anomalous samples of the input observation. However, the false rejection of some normal samples is possible.

The anomalousness criterion for a group of samples sounds as follows: a group is anomalous if it repeatedly occurs in rejected (inadmissible) pairs of grids and never occurs in admissible pairs of grids (those that have passed rejection).

In general, the ranking principle may lead to situations when the number of good grids satisfying condition (12) may increase, i.e., some redundancy of private estimates. Consequently, the following legitimate question arises: how should this redundancy be considered in order to construct a more reliable resulting estimate?

To construct the resulting estimate $\mathbf{A}^*$ of the vector $\mathbf{A}$, we proceed as follows. Let C_j^*, $j = \overline{1, J^*}$, denote all the grids included in the admissible pairs. (These grids will be called competing.) For each fixed grid C_j^* and all other competing grids C_m^*, we form the total scalar residual

$$\Delta_{\mathbf{A}j}^* = \sum_{\substack{m=1 \\ m \neq j}}^{J^*} \Delta_{\mathbf{A}jm}. \tag{13}$$

This residual shows the efficiency of the grid C_j^* compared to all other competing grids C_m^*, $m \neq j$, $m \in \overline{1, J^*}$, included in admissible pairs.

Obviously, within the models and constraints adopted, the criterion for selecting the optimal number $j^* \in \{1, \ldots, J^*\}$ of the optimal grid $\mathrm{C}_{j^*}^* \in \{\mathrm{C}_1^*, \ldots, \mathrm{C}_{J^*}^*\}$ is as follows:

$$j^* = \arg\min_j \Delta_{\mathbf{A}j}^*. \tag{14}$$

This criterion can be easily realized in practice if the matrix of scalar residuals is constructed for all admissible pairs of time grids. Among the input observation samples corresponding to the optimal grid C_{j*}^*, there are no anomalous samples and anomalous groups of irregular interference samples; in addition, the estimates

$$\begin{cases} \mathbf{A}_{j^*}^* = \mathbf{P}_{j^*}^{\mathbf{A}} \mathbf{H}_{j^*}, \\ s\left(t, \mathbf{A}_j^*\right) = \left(\mathbf{A}_{j^*}^*\right)^{\mathrm{T}} \boldsymbol{\Psi}(t) \end{cases} \tag{15}$$

will be GIUE-optimal for the grid $\mathrm{C}_{j^*}^*$.

Let us pass to subproblem 2. It is easily reduced to subproblem 1 by introducing the single vector of estimated parameters $\mathbf{Z} = \left[\mathbf{A}^{\mathrm{T}}, \mathbf{B}_\theta^{\mathrm{T}}\right]^{\mathrm{T}}$. Due to (9), we can assign to each grid C_j the GIUE-optimal estimate $\mathbf{Z}_j^* = \left[\left(\mathbf{A}_j^*\right)^{\mathrm{T}}, \left(\mathbf{B}_{\theta j}^*\right)^{\mathrm{T}}\right]^{\mathrm{T}}$. Finding the estimate $\mathbf{A}_j^*$ requires invariance with respect to the regular interference (i.e., the parameter $\boldsymbol{\Theta}_0$ in (1) is treated as a disturbing factor.) In turn, finding the estimate $\mathbf{B}_{\theta j}^*$ requires invariance with respect to the signal $\mathbf{S}_0$ (i.e., the signal now treated as a disturbing factor for the regular interference). The matrices $\mathbf{P}_j^{\mathbf{A}}$ and $\mathbf{P}_{\theta j}^{\mathbf{B}}$ are used to find the estimates $\mathbf{A}_j^*$ and $\mathbf{B}_{\theta j}^*$, respectively; they are formed by (10) and (11). Obviously, to solve subproblem 2, it suffices to replace $\Delta_{\mathbf{A}[v]}$, $\delta_{\mathbf{A}}$, $\Delta_{\mathbf{A}jm}$, and $\Delta_{\mathbf{A}j}^*$ in (12)–(14) with $\Delta_{\mathbf{Z}[v]}$, $\delta_{\mathbf{Z}}$, $\Delta_{\mathbf{Z}jm}$, and $\Delta_{\mathbf{Z}j}^*$, respectively.

For the grid $\mathrm{C}^*_{j^*}$, we obtain the following solution of the optimal regular interference identification problem:

$$\begin{cases} \mathbf{B}^*_{\theta j^*} = \mathbf{P}^{\mathbf{B}}_{\theta j^*}\mathbf{H}_{j^*}, \\ \theta\left(t, \mathbf{B}^*_{\theta j^*}\right) = \left(\mathbf{B}^*_{\theta j^*}\right)^{\mathrm{T}} \mathbf{\Omega}_\theta(t). \end{cases} \tag{16}$$

In view of (15) and (16), the final estimate for subproblem 2 is $\mathbf{Z}^*_{j^*} = \left[\left(\mathbf{A}^*_{j^*}\right)^{\mathrm{T}}, \left(\mathbf{B}^*_{\theta j^*}\right)^{\mathrm{T}}\right]^{\mathrm{T}}$.

For the method under consideration, it is fundamental to select competing grids C^*_j, $j = \overline{1, J^*}$, satisfying condition (12) and choose the optimal grid $\mathrm{C}^*_{j^*}$ among them. Obviously, this is directly related to the initial set of grids $\mathrm{C} = \{\mathrm{C}_0, \mathrm{C}_1, \ldots, \mathrm{C}_{J-1}\}$ satisfying conditions (2) and (3). Modern advances in the field of parallel computers (especially those on new principles [21–24]) give hope that the multiplication principle of grids and partial estimates will not become an obstacle for prospective real-time systems. However, this approach is not possible for all the existing systems, as it may require a huge number of channels for parallel data processing and significant computational costs. Therefore, along with the optimal solution, it is necessary to consider quasi-optimal approaches to the design of reduced grids.

4. SOME RULES FOR BUILDING REDUCED TIME GRIDS

According to the principle of grid multiplication, a set of reduced grids is formed on a given time interval by some rule χ,

$$\chi : \mathrm{C}_0 \to \{\mathrm{C}_j\}_{j=0}^{J-1}, \tag{17}$$

so that, under some constraints on irregular interferences, the resulting partial estimates $\left\{\mathbf{A}^*_j\right\}_{j=0}^{J-1}$ and $\left\{\mathbf{Z}^*_j\right\}_{j=0}^{J-1}$ based on such grids surely include not only bad but also good estimates for which condition (12) is valid. Obviously, in the general case, the choice of an appropriate rule χ is ambiguous and depends on a particular system and observation conditions.

Rule 1. Let the system operate a small sample. Using combinatorics and the basic grid C_0, we form all possible reduced grids with the number of nodes not less than $N_{\min}$. By the complete enumeration method, the number of such grids is equal to

$$J = \sum_{m=0}^{N_0 - N_{\min}} C_{N_0}^{N_{\min}+m}, \tag{18}$$

where $C_{N_0}^{N_{\min}+m}$ denotes the corresponding binomial coefficient (the number of combinations of $N_{\min} + m$ elements in a total of N_0 elements).

For a fixed M_d, the number of grids not associated with nonzero irregular interference samples is given by

$$\overline{J} = \sum_{m=0}^{\overline{N}} C_{N_0 - M_d}^{N_{\min}+m}, \tag{19}$$

where $\overline{N} = N_0 - N_{\min} - M_d$.

In the special case when $M_d = K_{\max}$, we can form the minimum number of such grids, $\overline{J}_{\min} = \sum_{m=0}^{\overline{N}_{\min}} C_{N_0 - K_{\max}}^{N_{\min}+m}$, where $\overline{N}_{\min} = N_0 - N_{\min} - K_{\max}$.

For example, let $N_0 = 5$, $N_{\min} = 3$, $K_{\max} = 2$, and $M_d = 1$; then we have $J = 16$, $\overline{J}_{\min} = 1$, and $\overline{J} = 5$. If an irregular interference sample (for the case $M_d = 1$) is anomalous, the boundedness condition (12) will hold for five grids and fail for eleven. Among the good grids, four will have a length of $N_j = 3$, $j = \overline{1,4}$, and the fifth grid a length of $N_5 = 4$.

Rule 2. For large samples, a more suitable rule χ is based on representing the grid C_0 as L elementary grids $\overline{C}_l$ adjacent to each other (the full cover method):

$$\mathrm{C}_0 = \bigcup_{l=1}^{L} \overline{\mathrm{C}}_l, \quad \overline{\mathrm{C}}_l \cap \overline{\mathrm{C}}_d = \varnothing, \quad \forall l \neq d \quad l, d \in \overline{1,L}, \tag{20}$$

where $\overline{\mathrm{C}}_l = \left\{t_{l_m}, m = \overline{1,M_l}\right\}$ is the elementary grid of nodes $t_{l_m} \in \mathrm{C}_0$, $l_m \in \overline{1,N_0}$, $\sum_{l=1}^{L} M_l = N_0$.

One should keep in mind that the grid $\overline{\mathrm{C}}_l$ matches only the neighbor nodes from the grid C_0, i.e., those following each other: $\left(t_{l_{m+1}} = t_{l_1+m}, m = \overline{0, M_l - 1}\right)$. It is easiest to consider the elementary grids $\overline{\mathrm{C}}_l$ of the same length by taking $M_l = M$ and $L \cdot M = N_0$.

From L elementary grids, using combinatorics, we can also form the desired family of different reduced grids $\left\{\mathrm{C}_j, j = \overline{1, J-1}\right\}$, with the number of nodes not less than $N_{\min}$. In contrast to Rule 1, now potential anomalousness is associated with individual elementary grids. Given the maximum possible number of elementary potentially anomalous grids, it is easy to find the total number of reduced grids not containing these regions and other characteristics of the rule χ.

Rule 3. Along with (20), the partial cover method can be used:

$$\mathrm{C}_0 \supset \bigcup_{l=1}^{L} \overline{\mathrm{C}}_l. \tag{21}$$

This method can minimize computations but involves possible errors in decision-making.

Rule 4. To avoid such errors, it is possible to realize the rule χ step by step using different sets of elementary grids at each step. In this case, we propose the following step-by-step methods of full and partial cover by elementary grids of length M_{li}:

$$\begin{cases} \mathrm{C}_{0i} = \bigcup\limits_{l=1}^{L_i} \overline{\mathrm{C}}_{li}, & i = \overline{1,I}, \\ \mathrm{C}_{0i} \supset \bigcup\limits_{l=1}^{L_i} \overline{\mathrm{C}}_{li}, & i = \overline{1,I}, \end{cases} \tag{22}$$

where i is the step number and $L_i > L$.

According to (22), at each step i we construct a family of grids C_i instead of C and check condition (12) for all its elements. If this condition completely fails, it is necessary to proceed to the next step. If this condition is valid at least for one reduced grid from C_I, this grid will be considered optimal. Obviously, the step-by-step method suits well for large grids C_0 owing to insignificant computational costs (compared to the complete enumeration method), but it may lose in terms of efficiency for large I. However, under conditions (2) and (3), such a situation is quite rare.

Rule 5. An even simpler way of building reduced grids is possible when the system includes an input observation analyzer to identify potentially anomalous regions on the segment $[0, T]$ (in terms of the presence of irregular interferences in them). By excluding the nodes corresponding to these regions from C_0, we can form the desired family of reduced grids satisfying conditions (2) and (3).

The choice of an appropriate rule to form reduced grids entirely depends on the system under consideration and the requirements imposed on it. Formulas (17)–(22) are quite convenient for the quantitative justification of reduced grids.

5. RECOGNITION ALGORITHMS UNDER UNCERTAINTY

The recognition algorithm for subproblem 1 includes the following steps.

Step 1.1. Based on C_0, construct the set of grids $\left\{\mathrm{C}_j, j=\overline{0,J-1}\right\}$.

Step 1.2. For each grid C_j, find the optimal estimation matrix $\mathbf{P}_j^{\mathbf{A}}$.

Step 1.3. For each grid C_j, find the estimate $\mathbf{A}_j^*=\mathbf{P}_j^{\mathbf{A}}\mathbf{H}_j$.

Step 1.4. For each pair C_j, C_m, form the scalar residual $\Delta_{\mathbf{A}jm}$.

Step 1.5. Based on all residuals $\Delta_{\mathbf{A}jm}$, construct the variational series $\Delta_{\mathbf{A}[v]}$, $v=\overline{1,(J^2-J)/2}$.

Step 1.6. Based on the variational series and condition (12), form the set of competing grids C_j^*, $j=\overline{1,J^*}$.

Step 1.7. For the competing grids, calculate the total residuals $\Delta_{\mathbf{A}j}^*$, $j=\overline{1,J^*}$.

Step 1.8. Based on the criterion (14), find the optimal number $j^*\in\{1,\ldots,J^*\}$ of the optimal grid $\mathrm{C}_{j^*}^*$.

Step 1.9. Using the matrix $\mathbf{P}_{j^*}^{\mathbf{A}}$ and the grid $\mathrm{C}_{j^*}^*$, calculate the estimate $\mathbf{A}_{j^*}^*$ for the vector $\mathbf{A}$ and the estimate $s\left(t,\mathbf{A}_{j^*}^*\right)=\left(\mathbf{A}_{j^*}^*\right)^{\mathrm{T}}\mathbf{\Psi}(t)$ for the useful signal $s\left(t,\mathbf{A}\right)$.

The recognition algorithm for subproblem 2 includes the following steps.

Step 2.1. Based on C_0, construct the set of grids $\left\{\mathrm{C}_j, j=\overline{0,J-1}\right\}$.

Step 2.2. For each grid C_j, find the optimal estimation matrices $\mathbf{P}_j^{\mathbf{A}}$ and $\mathbf{P}_{\theta j}^{\mathbf{B}}$.

Step 2.3. For each grid C_j, find the estimates $\mathbf{A}_j^*$ and $\mathbf{B}_{\theta j}^*$.

Step 2.4. Form the unified vector $\mathbf{Z}_j^*=\left[\left(\mathbf{A}_j^*\right)^{\mathrm{T}},\left(\mathbf{B}_{\theta j}^*\right)^{\mathrm{T}}\right]^{\mathrm{T}}$.

Step 2.5. For each pair C_j, C_m, form the scalar residual $\Delta_{\mathbf{Z}jm}$.

Step 2.6. Based on all residuals $\Delta_{\mathbf{Z}jm}$, construct the variational series $\Delta_{\mathbf{Z}j}^*$, $j=\overline{1,J^*}$.

Step 2.7. Based on the variational series and condition (12), form the set of competing grids C_j^*, $j=\overline{1,J^*}$.

Step 2.8. For the competing grids, calculate the total residuals $\Delta_{\mathbf{Z}j}^*$, $j=\overline{1,J^*}$.

Step 2.9. Based on the criterion (14), find the optimal number $j^*\in\{1,\ldots,J^*\}$ of the optimal grid $\mathrm{C}_{j^*}^*$.

Step 2.10. Using the matrices $\mathbf{P}_{j^*}^{\mathbf{A}}$ and $\mathbf{P}_{\theta j^*}^{\mathbf{B}}$ and the grid $\mathrm{C}_{j^*}^*$, calculate the estimate $\mathbf{A}_{j^*}^*$ for the vector $\mathbf{A}$ and the estimate $s\left(t,\mathbf{A}_{j^*}^*\right)=\left(\mathbf{A}_{j^*}^*\right)^{\mathrm{T}}\mathbf{\Psi}(t)$ for the useful signal $s(t,\mathbf{A})$, as well as the estimate $\mathbf{B}_{\theta j^*}^*$ for the vector $\mathbf{B}_\theta$ and the estimate $\theta\left(t,\mathbf{B}_{\theta j^*}^*\right)=\left(\mathbf{B}_{\theta j^*}^*\right)^{\mathrm{T}}\mathbf{\Omega}_\theta(t)$ for the regular interference $\theta(t,\mathbf{B}_\theta)$.

6. A COMPARATIVE ANALYSIS OF THE METHOD

The correlation matrices of the estimation errors on the grid $\mathrm{C}_{j^*}^*$ are found by the rule

$$\begin{cases}\mathbf{K}_{j^*}^{\mathbf{A}}=\mathbf{P}_{j^*}^{\mathbf{A}}\mathbf{K}_{\mathbf{\Xi}_{j^*}}\left(\mathbf{P}_{j^*}^{\mathbf{A}}\right)^{\mathrm{T}},\\ \mathbf{K}_{\theta j^*}^{\mathbf{B}}=\mathbf{P}_{\theta j^*}^{\mathbf{B}}\mathbf{K}_{\mathbf{\Xi}_{j^*}}\left(\mathbf{P}_{\theta j^*}^{\mathbf{B}}\right)^{\mathrm{T}}.\end{cases}\tag{23}$$

The expression (23) allows one to assess the potential capabilities of the method in each particular case, considering the requirements for the system.

According to the optimality criterion used, these matrices have minimum traces, i.e., $Sp\mathbf{K}_{j^*}^{\mathbf{A}}\to\min$ and $Sp\mathbf{K}_{\theta j^*}^{\mathbf{B}}\to\min$ in the class of all linear estimates. The methodological error

due to the finiteness of the representations (4) and (5) can be considered as follows. Let the observation equation on the optimal grid $\mathrm{C}^*_{j^*}$ have the form (with irregular interference compensation)

$$\mathbf{H}_{j^*} = (\mathbf{S}_{j^*} + \boldsymbol{\Delta}\mathbf{S}_{j^*}) + (\boldsymbol{\Theta}_{j^*} + \boldsymbol{\Delta}\boldsymbol{\Theta}_{j^*}) + \boldsymbol{\Xi}_{j^*}, \tag{24}$$

where $\boldsymbol{\Delta}\mathbf{S}_{j^*}$ and $\boldsymbol{\Delta}\boldsymbol{\Theta}_{j^*}$ are the additive corrections to the signal and regular interference due to the tails of the functional series used.

In this case, the estimate $\mathbf{A}^*_{j^*}$ of the vector $\mathbf{A}$ (calculated without the corrections) is given by

$$\mathbf{A}^*_{j^*} = \mathbf{P}^{\mathbf{A}}_{j^*}(\mathbf{S}_{j^*} + \boldsymbol{\Delta}\mathbf{S}_{j^*}) + \mathbf{P}^{\mathbf{A}}_{j^*}(\boldsymbol{\Theta}_{j^*} + \boldsymbol{\Delta}\boldsymbol{\Theta}_{j^*}) + \mathbf{P}^{\mathbf{A}}_{j^*}\boldsymbol{\Xi}_{j^*}, \tag{25}$$

where the optimal estimation matrix $\mathbf{P}^{\mathbf{A}}_{j^*}$ is found using the finite representations (4) and (5) by rejecting the tails.

Therefore, assuming $\boldsymbol{\Xi}_{j^*} = \mathbf{0}$, we have the following representation for the the true value of $\mathbf{A}$:

$$\mathbf{A} = (\mathbf{P}^{\mathbf{A}}_{j^*} + \boldsymbol{\Delta}\mathbf{P}^{\mathbf{A}}_{j^*})(\mathbf{S}_{j^*} + \boldsymbol{\Delta}\mathbf{S}_{j^*}) + (\mathbf{P}^{\mathbf{A}}_{j^*} + \boldsymbol{\Delta}\mathbf{P}^{\mathbf{A}}_{j^*})(\boldsymbol{\Theta}_{j^*} + \boldsymbol{\Delta}\boldsymbol{\Theta}_{j^*}). \tag{26}$$

The average value of the methodological error is

$$\overline{\Delta\mathbf{A}_{j^*}} = \mathrm{M}\left\{\mathbf{A} - \mathbf{A}^*_{j^*}\right\} = \boldsymbol{\Delta}\mathbf{P}^{\mathbf{A}}_{j^*}(\mathbf{S}_{j^*} + \boldsymbol{\Delta}\mathbf{S}_{j^*}) + \boldsymbol{\Delta}\mathbf{P}^{\mathbf{A}}_{j^*}(\boldsymbol{\Theta}_{j^*} + \boldsymbol{\Delta}\boldsymbol{\Theta}_{j^*}), \tag{27}$$

where $\mathrm{M}\{\cdot\}$ denotes the expectation operator under $\mathrm{M}\{\boldsymbol{\Xi}_{j^*}\} = \mathbf{0}$.

Formulas (23)–(27) can be used to select the necessary parameters of the method that minimize the resulting estimation error in each particular case. Necessary and sufficient conditions for the existence of a unique solution of the estimation problem within the method require nonsingularity and some constraints for the ranks of several matrices (by analogy with [18, 19]). In practice, the given conditions are satisfied by rationally choosing the functional bases used and the number of degrees of freedom in the signal and regular interference models as well as by setting up appropriate observation conditions. All these issues concern the planning of a computational experiment and are not considered further, as separate studies are required in each particular case.

To perform a comparative analysis, we consider several methods: $\mathrm{M}_{(1)}$ (XLS), $\mathrm{M}_{(2)}$ (GIUE), and $\mathrm{M}_{(3)}$ (the novel method). Each possible irregular interference can be assigned one of the hypotheses Γ_l, $l = \overline{1, L}$. Obviously, $L = J$ and, for fixed l, by analogy with (1) and (8), the model observation is

$$\mathbf{H}_{0l} = \mathbf{S}_0 + \boldsymbol{\Theta}_0 + \mathbf{D}_{0l} + \boldsymbol{\Xi}_0.$$

The recognition problem based on $\mathrm{M}_{(1)}$ is solved by minimizing the quadratic form $\chi(\mathbf{Z}_{dl})$:

$$(l^*, \mathbf{Z}^*_{dl}) = \arg\min_{l,\mathbf{Z}_{dl}} \chi(\mathbf{Z}_{dl}) = \arg\min_{l,\mathbf{Z}_{dl}} \left[\Delta(l, \mathbf{Z}_{dl})\right]^{\mathrm{T}} (\mathbf{K}_{\boldsymbol{\Xi}_0})^{-1} \left[\Delta(l, \mathbf{Z}_{dl})\right], \quad i \in \{1, 2, 3\}, \tag{28}$$

where $\Delta(l, \mathbf{Z}_{dl}) = \mathbf{H}_0 - \mathbf{H}_{0l}$, $\mathbf{H}_{0l} = \mathbf{H}_{0l}(l, \mathbf{Z}_{dl})$, and $\mathbf{Z}_{dl} = \left[\mathbf{Z}^{\mathrm{T}}, \mathbf{B}^{\mathrm{T}}_{dl}\right]^{\mathrm{T}} = \left[\mathbf{A}^{\mathrm{T}}, \mathbf{B}^{\mathrm{T}}_{\theta}, \mathbf{B}^{\mathrm{T}}_{dl}\right]^{\mathrm{T}}$.

One measure of the efficiency of $\mathrm{M}_{(i)}$ is the dimension of the matrices under inversion. Clearly, for a fixed l, $\mathrm{M}_{(1)}$ requires to invert a matrix of order $\rho_{(1)l} = M_s + M_\theta + M_{dl}$, where M_{dl} is the number of nonzero coordinates in the vector $\mathbf{D}_{0l}$. In turn, $\mathrm{M}_{(2)}$ implies the formation of a joint basis matrix for the regular and irregular interferences; for a fixed l, this leads to the inversion of two matrices of order $\rho_{(2)s} = M_s$ and $\rho_{(2)\theta dl} = M_\theta + M_{dl}$, respectively. Regardless of the number of the hypothesis under consideration, $\mathrm{M}_{(3)}$ requires the inversion of two matrices of orders $\rho_{(3)s} = \rho_{(2)s} = M_s$ and $\rho_{(3)\theta} = M_\theta$. Therefore, we can choose the most preferable method by the criterion

$$i^* = \arg\min_i \rho_{(i)},$$

where $i, i^* \in \{1, 2, 3\}$, $\rho_{(1)} = \max\limits_l\{M_s + M_\theta + M_{dl}\}$, $\rho_{(2)} = \max\limits_l\{M_s, M_\theta + M_{dl}\}$, and $\rho_{(3)} = \max\limits_l\{M_s, M_\theta\}$.

As can be seen, in all the cases, $\rho_{(3)} \leqslant \rho_{(1)}$ and $\rho_{(3)} \leqslant \rho_{(2)}$; in the absence of irregular interference (but under regular interference), we have $\rho_{(3)} < \rho_{(1)}$ and $\rho_{(3)} = \rho_{(2)}$. Without regular and irregular interferences, $\rho_{(1)} = \rho_{(2)} = \rho_{(3)}$.

The analysis shows that if the dimension of the regular and irregular interferences is high, the condition $M_\theta + M_{dl} > M_s$ is valid, and the matrices under inversion are ill-conditioned, $\mathrm{M}_{(3)}$ will be much preferable to $\mathrm{M}_{(1)}$ and $\mathrm{M}_{(2)}$ in terms of computational stability. (In this case, we always have $\rho_{(3)} < \rho_{(1)}$ and $\rho_{(3)} < \rho_{(2)}$). For these conditions, along with $\rho_{(1)}$, $\rho_{(2)}$, and $\rho_{(3)}$, the conditionality numbers $\mu_{(1)}$, $\mu_{(2)}$, and $\mu_{(3)}$ must be used, which are the stability characteristics of the methods compared.

To comparatively assess the computational complexity of the methods, we will use the following characteristics: $V_{(i)}^{\Sigma}$ (the total number of nodes of the original grid C_0 that are used to test all hypotheses); $Q_{(i)}^{\Sigma}$ and $T_{(i)}^{\Sigma}$ (the total number of operations (addition and multiplication) and time, respectively, required to realize $\mathrm{M}_{(i)}$.) The following characteristics are adopted for comparative accuracy assessment: $\Delta s_{(i)}$ and $\Delta\theta_{(i)}$ (the resulting estimation errors of the signal and regular interference, respectively). They are given by $\Delta s_{(i)} = \max\limits_t \left| s(t, \mathbf{A}) - s\left(t, \mathbf{A}_{(i)}^*\right)\right|$ and $\Delta\theta_{(i)} = \max\limits_t \left|\theta(t, B_\theta) - \theta\left(t, B_{\theta(i)}^*\right)\right|$.

Omitting intermediate calculations, we present the final results:

— for the characteristic $V_{(i)}^{\Sigma}$,

$$V_{(1)}^{\Sigma} = V_{(2)}^{\Sigma} = LN_0, \quad V_{(3)}^{\Sigma} = \sum_{m=0}^{N_0 - N_{\min}} C_{N_0}^{N_{\min}+m}\left(N_0 - m\right);$$

— for the characteristic $Q_{(i)}^{\Sigma}$,

$$Q_{(1)}^{\Sigma} = \sum_{m=0}^{N_0 - N_{\min}} \left\{ C_{N_0}^{N_{\min}+m}\left[2N_0^3 + 6N_0^2\gamma_m + N_0\left(4\gamma_m^2 - 3\gamma_m\right) + 2\gamma_m^3 - \gamma_m^2\right]\right\},$$

$$\gamma_m = M_s + M_\theta + m,$$

$$Q_{(2)}^{\Sigma} = \sum_{m=0}^{N_0 - N_{\min}} \left\{ C_{N_0}^{N_{\min}+m}\left[2(N_0^3 + N_0^2\varphi_m) + N_0\left(4\beta_m - \varphi_m\right) + 2(\alpha_m - \varphi_m)\right]\right\},$$

$$\beta_m = \eta_m^2 + M_s^2, \quad \alpha_m = \eta_m^3 + M_s^3, \quad \varphi_m = 2\eta_m + 3M_s, \quad \eta_m = M_\theta + m,$$

$$Q_{(3)}^{\Sigma} = \sum_{m=0}^{N_0 - N_{\min}} \left\{ C_{N_0}^{N_{\min}+m}\left[2\left(N_0 - m\right)^3 + 2\left(N_0 - m\right)^2\varphi_0 + \left(N_0 - m\right)\left(4\beta_0 - \varphi_0\right) + 2\alpha_0 - \beta_0\right]\right\},$$

$$\alpha_0 = (M_\theta + M_d)^3 + M_s^3, \quad \beta_0 = M_\theta^2 + M_s^2, \quad \varphi_0 = 2M_\theta + 3M_s, \quad \eta_0 = M_\theta.$$

According to the analysis of these expressions, a significant computational gain $\mathrm{M}_{(3)}$ over $\mathrm{M}_{(1)}$ and $\mathrm{M}_{(2)}$ can be achieved by processing the data on reduced grids of smaller volume (compared to the original grid C_0). This gain also grows with increasing the dimension of the irregular interference.

The calculation of the characteristics $T_{(i)}^{\Sigma}$ and $\Delta\theta_{(i)}^{\Sigma}$ depends on both the initial data of the problem and the capabilities of the computing environment used.

To test the family of hypotheses, L parallel data processing channels can be organized to perform a large amount of vector-matrix computations. However, significant optimization of such computations is possible since all vectors and matrices used in different channels are obtained by

an appropriate reduction of the vectors and matrices on the basic grid C_0. In other words, most operations in different channels are repeated.

The novel method can be easily extended to other problems (e.g., detection and discrimination) related to signal recognition (including those with nonlinear parameters [19]) and solved within the theory of hypotheses. In this case, the number of parallel data processing channels may increase significantly, as the reduced grids should be constructed directly for each hypothesis and each node of the definitional domain of a nonlinear parameter. Here, the most effective approaches to deal with irregular interferences are the full (20) and partial (21) cover methods, e.g., step-by-step (22).

7. AN ILLUSTRATIVE EXAMPLE

We apply the novel method to the single-position passive ranging of a radiating target with partially known motion parameters only by energy measurements (the energy method [24, 25]). Consider a target moving in the direction toward a rangefinder so that the slant range varies according to the law $R(t) = R_0 + R_0^{(1)}t + 2^{-1}R_0^{(2)}t^2$, where R_0, $R_0^{(1)}$, and $R_0^{(2)}$ are the initial range, radial velocity, and radial acceleration, respectively, before the time instant $t = 0$. Let the measured signal (energy parameter) be $s(t) = 1 - q_0^{-1}(t)$, where $q_0(t) = \left[p_0^{-1}p(t)\right]^{1/2}$ and $p(t)$ is the power of the electromagnetic wave at the rangefinder's input, $p_0 = p(0)$.

The well-known passive location equation for a stationary channel has the form

$$p = \zeta_0 R^{-2},$$

where $p = p(t)$, $R = R(t)$, and $\zeta_0 = \mathrm{const}$ is a generalized coefficient relating power and range.

Such a simplified model is encountered in practice under several constraints on the observation conditions of a radiating target [24, 25]. In the case considered here, it will demonstrate well the effectiveness of the novel method without resorting to complex ranging algorithms for the nonstationary case.

The required range is given by

$$R(t) = s^{-1}(t)D_0(t)[1 - s(t)], \quad t > 0, \tag{29}$$

where $D_0(t) = \left(R_0^{(1)}t + 2^{-1}R_0^{(2)}t^2\right)$ means the distance traveled by the target in the time t.

Formula (29) serves to calculate the range based on estimates of the parameter $s(t) = 1 - q_0^{-1}(t)$. Obviously, $s(t)$ can be represented as

$$s(t) = -\left(R_0^{-1}R_0^{(1)}t + 2^{-1}R_0^{-1}R_0^{(2)}t^2\right), \quad t > 0.$$

In view of (4), it follows that $s(t) = s(t, a_1, a_2) = a_1\psi_1(t) + a_2\psi_2(t)$, i.e., $M_s = 2$, $a_1 = -R_0^{-1}R_0^{(1)}$, $a_2 = -2^{-1}R_0^{-1}R_0^{(2)}$, $\psi_1(t) = t$, and $\psi_2(t) = t^2$. Thus, after estimating the coefficients a_1 and a_2 from measurements of the energy parameter $s(t, a_1, a_2)$, we can find the desired range by formula (29). Next, $\mathrm{M}_{(3)}$ will be used for estimation whereas $\mathrm{M}_{(1)}$ and $\mathrm{M}_{(2)}$ for comparative analysis. Note that all time parameters below are measured in seconds, distances in meters, radial velocity in meters per second, radial acceleration in meters per second squared, and the sample clogging factor in percent. The energy parameter is a dimensionless quantity.

For the experiment, we took $R_0 = 3 \times 10^4$, $R_0^{(1)} = 2 \times 10^3$, $R_0^{(2)} = 2 \times 10^2$, $T = 10$, $N_0 = 10$, $t_n - t_{n-1} = \Delta t = 1$, $t_1 = 1$, $t_{10} = 10$, $M_d = K_{\max} = 2$, $N_{\min} = 6$, $k_d = 20$, and $M_\theta = 0$. (In other words, the influence of the regular interference was neglected.) The measurement noise was modeled using a random number generator of the Gaussian distribution with the diagonal matrix $\mathbf{K}_{\boldsymbol{\Xi}_0}$ with the nonzero element $\sigma_0^2 = 10^{-10}$. For the cases when $N \geqslant N_{\min}$, and in the absence of an

irregular interference, the average root-mean-square error of range estimation (over an ensemble of 500 realizations) by any method must satisfy the condition $\overline{\Delta R} \leqslant 5 \times 10^2$.

The computations were carried out in MATLAB ver. R20119b on a PC with a 2.6 GHz quad-core CPU and 8 GB DDR3 RAM.

Table 1 shows the exact values of the energy parameter on the grid C_0.

Table 1

t_n	1	2	3	4	5	6	7	8	9	10
s_n	0.067	0.147	0.230	0.320	0.417	0.520	0.630	0.747	0.870	1

The following intermediate values were calculated for the initial data: $L = J = 386$, $\gamma_m = 2 + m$, $\beta_m = m^2 + 4$, $\alpha_m = m^3 + 8$, $\varphi_m = 2m + 6$, and $\eta_m = m$, $0 \leqslant m \leqslant 4$.

The final results of the computational experiment (after rounding) are combined in Table 2.

Table 2

Characteristics / $\mathrm{M}_{(i)}$	$\rho_{(i)}$	$\mu_{(i)}$	$V^{\Sigma}_{(i)}$	$Q^{\Sigma}_{(i)}$	$T^{\Sigma}_{(i)}$	$\overline{\Delta R_{(i)}}$
$\mathrm{M}_{(1)}$	6	544	3.9×10^3	2.5×10^6	1.3×10^{-3}	1.3×10^3
$\mathrm{M}_{(2)}$	4	148	3.9×10^3	1.9×10^6	9.9×10^{-4}	9.3×10^2
$\mathrm{M}_{(3)}$	2	103	2.6×10^3	4.7×10^5	2.4×10^{-4}	8.2×10^2

According to the data of this table, XLS and GIUE lose to the novel method in all their characteristics even under a small dimension of the matrices inverted. Using the result of [25], we can consider the nonstationary case ($\zeta_0 \neq \mathrm{const}$) with not only irregular interferences but also regular ones. In such conditions, the vector of estimated parameters expands, which requires increasing the volume of the original grid C_0. The effect achieved grows significantly with increasing the dimension of the problem and this volume.

The ranging method considered in this example, in combination with the algorithm proposed for estimating the energy parameter (based on the novel method), can be used both independently and with other known passive location methods. The former option can be appropriate when the energy channel is the only source of reliable information in the rangefinder (e.g., under either the single-position energy design of the rangefinder or the structural degradation of the two- or multi-position system: failures of separate positions, the abnormal operation of angular measurement channels, communication lines, etc.). The capability to solve the ranging problem autonomously with both regular and irregular interferences can find a wide application in various fields of passive location systems.

The latter option allows treating the passive-energy method as an alternative method when constructing a complex algorithm for the reliable operation of an intelligent system under various, even unfavorable, conditions. In addition to the novel method, such an algorithm can be based on the angular-measuring energy method and passive methods widely used in practice (difference ranging, triangulation, etc. as well as their different modifications).

Energy measurements traditionally do not belong to the class of reliable measurements. Hence, the novel signal recognition method under regular and irregular interferences may improve the operating efficiency of energy channels with the required accuracy of amplitude and power measurements.

8. CONCLUSIONS

The signal recognition method developed in this paper realizes the principles of continuity, invariance, multiplication, and ranking using a family of reduced grids. It can be effectively combined

with algorithms for orthogonal decomposition and solving ill-posed problems [27–29]. The possibility of decomposing computational procedures, reducing the dimension of matrices inverted, and decreasing the amount of computations allows solving more efficiently a whole range of applications-oriented problems with measurement processing in various fields. The signal recognition algorithms under regular and irregular interferences are not difficult to implement in special-purpose multichannel computers for systems operating in real time.

Compact analytical expressions have been derived to select in advance, for a particular application, the necessary models of signals and interferences as well as the values of their parameters to achieve the potential capabilities of the novel method. All computational procedures in each channel are reduced to the simplest mathematical operations over vectors and matrices; it is possible to combine this method with traditional approaches to solving applied problems on the optimal and quasi-optimal processing of measurements.

Advances in parallel computing give hope that any problems related to signal recognition can soon be solved using the novel method.

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